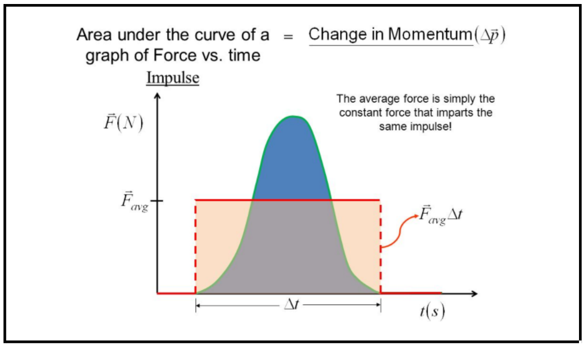
Theoretically modelling a projectile's force of impact with respect to distance

Personal Engagement

I have always been aware of the forces that govern our day to day lives. In taking IB Physics HL, I have not only been made aware of these forces, but I've also come to understand the subtle negotiations that take place between agents in an environment and the environment itself. When I was younger, I recall asking "Santa" for a NERF gun to play with. I can distinctly remember when NERF began introducing their electric powered models such as the Nerf N-Strike HyperFire Blaster. "Santa", however, decided that it would be better for me to play with the spring powered NERF Elite Firestrike. This fascination lasted well into my early teenage years, until I stopped playing with them because the spring's strength began to decay from years of use. The spring's decay resulted in the foam bullet's loss of 'impact', and an observed decrease in overall range.

Although the spring's inability to compress and expand to its maximum amplitude no doubt led to a decrease in the foam bullet's 'impact' and range, armed with a better understanding of physics, I now find myself questioning the relationship between the projectile's range and the resultant force of impact.



F1.a (Force v. Time Graph)

The foam bullet's force as it makes contact with a surface is found by integrating the function F(t) as shown by the graph above. The graph, however, fails to explicitly demonstrate the effect that distance has on final FoI (force of impact). This begged the question, "How does the distance travelled affect the force of impact that one feels when hit with a foam bullet?". The following relationship can be modelled mathematically:

General Equation of Parabola with respect to max force, F_m, and duration of impact, d_i:

$$F(t) \; = \; -\frac{4F_m}{d_i}t(-\frac{t}{d_i} + 1)$$

Indefinite Integral of "General Equation of Parabola with respect to F_m and d_i":

$$\int F(t)\,dt \; = \frac{F_m t^2}{d_i}(2 - \frac{4t}{3d_i}) + C$$

Force of Impact for a given duration of impact d_i and max force F_m (discussed later):

$$\int_0^{d_i} F(t)\,dx \; = \frac{2}{3}F_m d_i$$

This relationship can be used to explore far more questions than, "Why does it hurt more when a NERF gun is shot at point blank than when I am six feet away?". This relationship could also be used to explore the necessary thrust required for a water rocket to take off, or the force exerted from a dentist's water syringe at distance x. I designed a lab that would measure the force of impact from a foam bullet at 10 different distances, each 1 cm apart, in order to determine why there is variation between force of impact in relation to its distance. Furthermore, I'd like to propose a model that takes into account these subtle negotiations that take place between the foam bullet, the air, and losses of energy as it approaches its final destination.

Exploration

Experimental Question: At what rate does a foam bullet's force of impact change in respect to its total displacement from origin?

Hypothesis: I expect the foam bullet's force of impact to decrease in a negative exponential function as the distance increases, because of losses in energy from the friction between the foam bullet and the nozzle as well as friction from the air. Moreover, I expect this variation to be reproducible for similar model NERF guns.

Background Information:

> "The rate of change of momentum of an object is directly proportional to the force applied, and this change takes place in the direction of the applied force"
>
> - Isaac Newton, *Principiæ Naturalis Principia Mathematica*, 1687

Newton's Second Law of Motion, as demonstrated by an excerpt from Principiæ Naturalis Principia Mathematica, describes scenarios such as how hard a car will impact a wall, the pull that two planets experience, and, more importantly, a foam bullet's FoI. The mathematical notation for Newton's second law and my proposed derivation for theoretical force of impact are both shown below:

$F = ma$	**(F.1) [Force Formula]**
$\int_0^{d_i} F(t)\,dx = \frac{2}{3}F_m d_i$	**(F.2) [Proposed FoI]**

Comparing Theoretical FoI **F.2** and Newton's 2^{nd} law **F.1**, it is evident that **F.2** is composed of **F.1**; as denoted by max force, F_m. The FoI is defined as being impulse, J. It's relation to force of impact is demonstrated through the derivation below:

- $F = ma$

⇔ {restate a in respect to velocity and time $\frac{\Delta v}{\Delta t}$}

$$F = m\frac{\Delta v}{\Delta t}$$

⇔ {restate F in respect to momentum p}

$$p = m\Delta v$$

$$F = m\frac{\Delta v}{\Delta t} = \frac{\Delta p}{\Delta t}$$

⇔ {restate F in respect to impulse J}

$$J = \Delta p$$

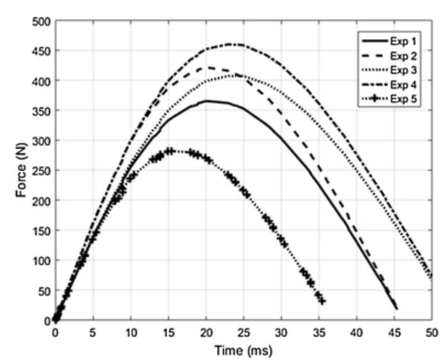
$$F = \frac{\Delta p}{\Delta t} = \frac{J}{\Delta t}$$

⇔ {solve for J}

$$J = F\Delta t$$

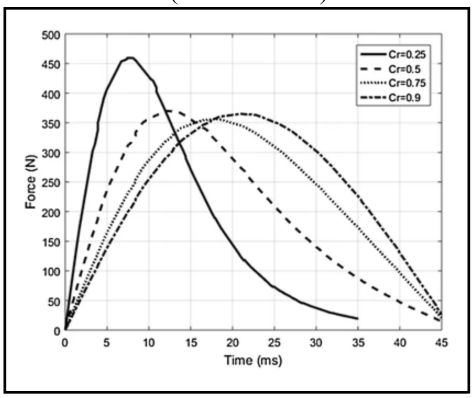
Where J represents FoI and F represents the Force exerted for time period Δt. In a graph where the y-axis represents force and the x axis represents time, taking the integral of the graph's function yields the area under the curve; which is a restatement of $F\Delta t$. FoI can also be calculated using work and energy, but this method requires knowing the speed of the object. Directly measuring the foam bullet's FoI, given the available resources, is easier than measuring the foam bullet's velocity in a way that doesn't obstruct the foam bullet. As such, the primary method will be graphing the instantaneous force readings from the load cell in respect to time to generate a $F(t)$ function.

Modelling Theoretical Impact Force:
Researching FoI, it became clear that different scenarios required different types of models. The $F(t)$ graph of drumsticks on a drum would look different than the $F(t)$ graph of a car collision. Because $F(t)$ graphs are usually highly nonlinear, creating a model for theoretical impact force is highly dependent on the shape of the expected data. In a paper assessing "power- and force-limited collaborative operation" in industrial robots, the $F(t)$ graphs that were modelled are similar to parabolic curves (shown below).



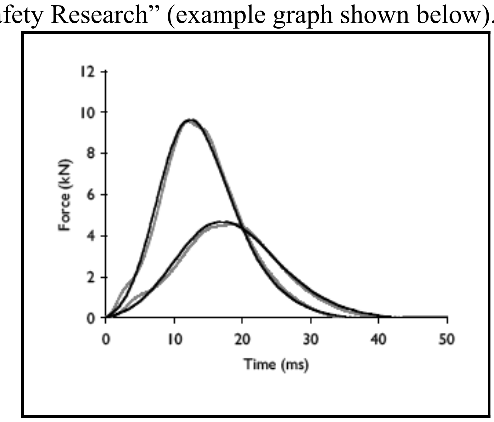
(I.2 Theoretical $F(t)$ graph)

The authors of the paper make note that, “considering the limitations in the currently available analytical modeling approaches… to realistically represent the highly nonlinear, isotropic, and non-homogenous nature of the physical collisions, this deviation of the model can be considered reasonable”. The authors make reference to a deviation in the parabolic model and the actual data (shown below)



(I.3 Actual $F(t)$ graph)

Similar approaches for modelling the FoI as parabolic-like curves are demonstrated by the University of Adelaide’s “Centre for Automotive Safety Research” (example graph shown below).



(I.4 Centre for Automotive Research $F(t)$ graph)

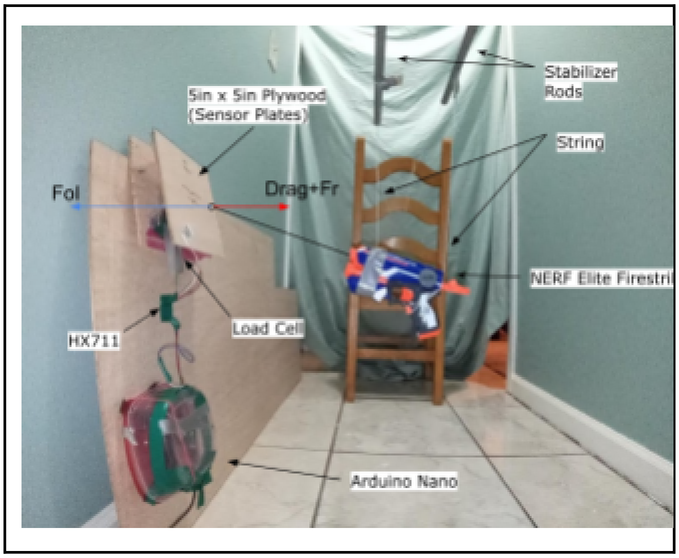
Because of its abundance in research papers, I have chosen to model my theoretical force of impact as a parabolic curve. Although this may not be an accurate representation of the NERF gun’s impact distribution, it is an assumption made on the basis that current literature utilizes the same form of model. Furthermore, I am making the assumption that the foam bullet impacts are more likely to be somewhSat symmetrical as opposed to a 3, 4, or even 5th degree polynomial.

Materials:

The materials required for this experiment are:

- Hx711 Load Cell Amplifier (x1)
    - Used to, as the name would suggest, amplify the signal received from the load cell so it can be read from the Arduino Nano.
- 1kg Load Cell (x1)
    - A load cell is a force transducer. It converts a force such as tension, compression, pressure, or torque, into an electrical signal that can be measured and standardized. As the force applied to the load cell increases, the electrical signal changes in a measurable manner.
- Arduino Nano (x1)
    - Used to interpret raw data from the load cell for data collection.
- Nerf Gun (NERF Elite Firestrike) (x1)
    - The model of the NERF gun is not relevant to the experiment so long as consistency is maintained throughout trials.
- Raspberry Pi (x1) [Optional]
    - The Raspberry Pi receives data from the Arduino Nano and then records it onto a file for further analysis (Any computer can work).
- 5in x 5in (0.5cm thick) plywood (x2)
- Approximately 1.60m of string (can be sewing string, kevlar string, etc) (x1)
- 2 rods of approximately the same size (used to stabilize the NERF gun)
    - Can be steel, aluminum, plastic, wood, PVC, etc. They are there to simply stabilize the NERF gun with strings.
- Meter Stick / Tape measure (x1)

*Free Body Diagram**:



(I5 Free Body Diagram)

* The meter stick is not shown in the Free Body Diagram, but should be included when setting up the experiment!

Methodology:

I. Begin by cutting two 80cm pieces of string and tying them on opposite ends of the NERF gun.
   A. These pieces will be used to stabilize the NERF gun on the stabilizer rods. They keep the angle of the nerf gun, and it's position, constant when performing the experiments.
   B. Make sure that the string's placement, when tied to the NERF gun, will not interfere with the foam bullet's path.

II. Once the string has been tied to the NERF gun and they've been secured to the stabilizer rack (at your preferred height), ensure the string will not move by placing tape on top of the string.

III. Using the load cell and two wooden 5in x 5in cut-outs, properly assemble the sensors.

IV. Secure the load cell sensor to a platform (such as, but not limited to, a wall) such that the foam bullet's impact is approximately perpendicular to the sensor's platform.

V. Connect the Load Cell to the HX711 Load Cell Amplifier, and then connect the HX711 Load Cell Amplifier to their corresponding Arduino pins.

VI. Plug in the arduino to your computer and run the "FDCR.py" and "FDCU.py" programs that can be found on the following [github repository](#).
   A. You will need to install the following libraries to run the programs: MatplotLib and Pyserial.

VII. Once running, head over to "FDCR.py" and select the "LivePreview Page".
   A. The data is streamed in real-time from the device running "FDCU.py" and can also be locally saved to specific folders for later analysis.

VIII. Lightly touch the load cell sensor and ensure that it is sending data to the computer. If there is a change in the displayed graph, the data is being transmitted.

IX. Load the NERF gun with the foam bullets

X. Measure out distance from the load cell sensor and change the distance accordingly.

XI. Fire the NERF gun.

XII. Save the data.
   A. This can be done by simply writing a file name / destination folder in the corresponding boxes inside of "FDCR.py" and clicking the "save data" button.

XIII. Repeat from IX. until all data points have been recorded.

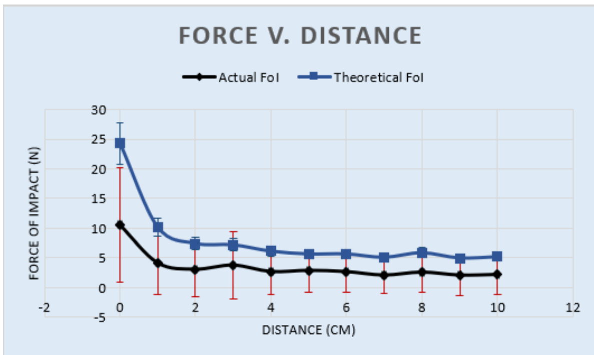
XIV. When done, simply disconnect the sensor from the computer (or device streaming the data) and disassemble the remaining parts of the lab.

Variables:

Variable Name	Symbol	Unit	Equation Reference
Force	F	N	$F = ma$
Mass	m	kg	$F = ma$
Time	Δt	s	$F = ma$
Impulse	J	N	$J = Ft$
Duration of Impact	d_i	s	$F = \frac{\Delta p}{d_i}$

Maximum Force	F_m	N	$F(t) = -\frac{4F_m}{d_i}$

Evaluation



(R1 Final $F(t)$ Graph)

Distance (±0.005)	Actual FoI (N)		Theoretical FoI (N)	
0	10.6	±9.60	24.3	±3.54
1	4.17	±5.40	10.2	±1.49
2	3.10	±4.60	7.44	±1.08
3	3.79	±5.60	7.20	±1.05
4	2.70	±3.90	6.13	±0.894
5	2.90	±3.60	5.66	±0.825
6	2.69	±3.50	5.67	±0.827
7	2.10	±3.10	5.10	±0.738
8	2.60	±3.40	5.86	±0.855
9	2.10	±3.40	4.99	±0.728
10	2.20	±3.30	5.20	±0.758

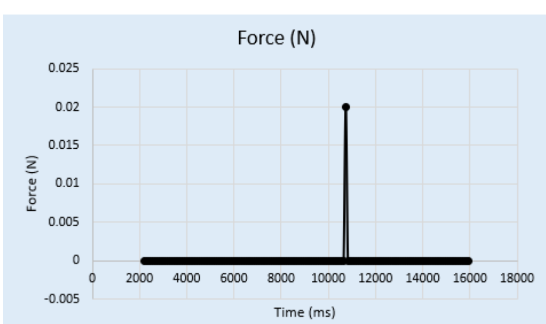
(R2 Force Distance Chart)

A glimpse of the results

The final Force v. Distance graph and its corresponding table are shown above. The results will be explained in further detail and are provided as a reference for understanding the following explanations.

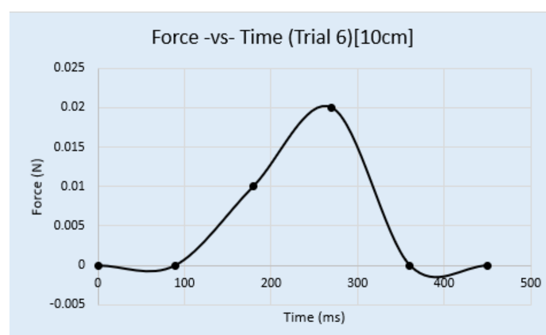
Data Processing

The experiment relied on the Arduino Nano's ability to record data within 90ms intervals. The duration of impact for most objects is characteristically fast and very hard to measure using off-the-shelf stopwatches or timers. Even with the Arduino Nano collecting data every 90ms, the $F(t)$ graphs for this experiment were only able to capture a very 'rough' image of the foam bullet's impact with the load cell sensor.



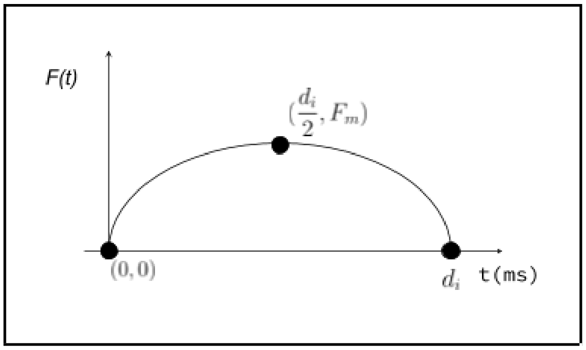
(R3 Raw $F(t)$ for Trial 8 at 10cm)

When analyzing the foam bullet's impact, I limited the scope of the data to include only data points relating to the impact. These points were also made relative to zero such that the first data point occurred at (0, 0).



(R4 Relativized $F(t)$ for Trial 6 at 10cm)

Making the first moment of impact (0, 0) was crucial to making the theoretical FoI formula. The graph below provides a symbolic representation of a theoretical $F(t)$ graph:



(R5 Symbolic Representation of FoI Parabola)

Deriving the Indefinite Integral for FoI

The relationship between $F(t)$ and Impulse was explained briefly in the "Exploration" section of the IA and is expanded here. In order to calculate the Impulse, otherwise known as FoI, the $F(t)$ must first be integrated. Below is the derivation for the formula:

Indefinite Integral

Finding General Equation of Parabola with respect to F_m and d_i

$$F(t) = -a(x-h)^2 + k$$

$$F(t) = -a(t - \frac{d_i}{2})^2 + F_m$$

$$F(t) = -a(t - \frac{d_i}{2})(t - \frac{d_i}{2}) + F_m$$

$$F(t) = (-at + \frac{ad_i}{2})(t - \frac{d_i}{2}) + F_m$$

$$F(t) = -at^2 + \frac{ad_i t}{2} + \frac{ad_i t}{2} - \frac{ad_i^2}{4} + F_m$$

$$F(t) = -at^2 + 2(\frac{ad_i t}{2}) - \frac{ad_i^2}{4} + F_m$$

$$F(t) = -at^2 + ad_i t - \frac{ad_i^2}{4} + F_m$$

Assuming t is 0 and $F(0)$=0…

$$F(0) = -a(0)^2 + ad_i(0) - \frac{ad_i^2}{4} + F_m$$

$$0 = -\frac{ad_i^2}{4} + F_m$$

$$-F_m = -\frac{ad_i^2}{4}$$

$$\frac{4F_m}{d_i^2} = a$$

Replacing coefficient "a" into general equation

$$F(t) = -\frac{4F_m}{d_i^2}t^2 + \frac{4F_m}{d_i^2}d_i t - \frac{4F_m}{d_i^2}\frac{d_i^2}{4} + F_m$$

$$F(t) = -\frac{4F_m}{d_i^2}t^2 + \frac{4F_m}{d_i^2}t - F_m + F_m$$

$$F(t) = -\frac{4F_m}{d_i^2}t^2 + \frac{4F_m}{d_i^2}t$$

$$F(t) = -\frac{4F_m}{d_i}t(-\frac{t}{d_i} + 1)$$

Indefinite Integral

$$\int F(t)\,dt = -\frac{4F_m t^3}{3d_i^2} + \frac{4F_m t^2}{2d_i} + C$$

$$\int F(t)\,dt = -\frac{4F_m t^3}{3d_i^2} + 2\frac{F_m t^2}{d_i} + C$$

$$\int F(t)\,dt = \frac{F_m t^2}{d_i}(2 - \frac{4t}{3d_i}) + C$$

Definite Integral

Using the indefinite integral, we can derive a general equation for finding the integral between points 0, and d_i as follows:

$$\int_0^{d_i} F(t)\,dt = (\frac{F_m t^2}{d_i}(2 - \frac{4t}{3d_i}) + C) - (\frac{F_m t^2}{d_i}(2 - \frac{4t}{3d_i}) + C)$$

$$\int_0^{d_i} F(t)\,dt = (\frac{F_m d_i^2}{d_i}(2 - \frac{4d_i}{3d_i}) + C) - (0 + C)$$

$$\int_0^{d_i} F(t)\,dt = F_m d_i(2 - \frac{4}{3}) + C - C$$

$$\int_0^{d_i} F(t)\,dt = F_m d_i(2 - \frac{4}{3})$$

$$\int_0^{d_i} F(t)\,dt = \frac{2}{3}F_m d_i$$

"$\int_0^{d_i} F(t)\,dt = \frac{2}{3}F_m d_i$", describes the relationship between the maximum force, F_m, and duration of the impact, d_i, and theoretical FoI. For a distance, x, there is a corresponding theoretical and actual FoI. The theoretical force of impact was calculated by finding the maximum force of an impact and it's duration and integrating it using the definite integral of the function $F(t)$.

Example Calculations (TFoI)

(R6 $F(t)$ graph for 9th trial at 10cm displacement)

From the graph above, we can extract the following information:

F_{max} →	0.01N
d_i →	449ms
F_{avg} →	0.003N

Using this information, we can calculate the theoretical FoI:

$$\int_0^{449ms} F(t)\,dt = \frac{2}{3}F_m d_i + C$$

$$\int_0^{449ms} F(t)\,dt = \frac{2}{3}(0.01N)(449ms) + C - \frac{2}{3}(0N)(0$$

$$\int_0^{449ms} F(t)\,dt = \frac{2}{3}(0.01N)(449ms)$$

$$\int_0^{449ms} F(t)\,dt = 2.993 \approx 3.0\ N$$

<u>Theoretical FoI</u>: $3.0\ N$

As for the actual FoI, due to the simplistic geometric nature of the data points, I wrote a simple python script to calculate the integral using rectangles and triangles. The manual integration is shown in summation notation below:

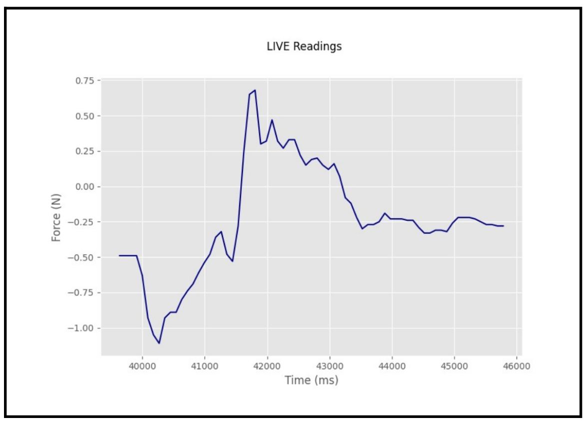
$$\sum_{n=0}^{k=\frac{90}{d_i}} F(90n)90 \bullet \left(1 + \frac{F(90(n+1)) - 1}{2}\right)$$

Where $n = 0$ represents the first "segment" of data, $k = \frac{90}{d_i}$ represents the number of "segments" in a certain time period d_i, $F(90n)90$ represents the area of a rectangle and $\frac{F(90(n+1))-1}{2}$ represents the area of the triangle above the rectangle for that same "segment". In the case for the trial above, the AFoI is shown below:

<u>Actual FoI</u>: $0.895\ N$

Observations for F(t) Data Collection

When I first began testing the load cell, I did so by placing a 1kg weight on top of the sensing platform. The resultant graph's shape was very reminiscent of rapidly changing slopes. This preliminary testing of the equipment guided my experimentation and led me to believe that the foam bullet's $F(t)$ graphs would be difficult to integrate using normal geometric shapes.



(R7 Example Readings from Load Cell)

During data collection, however, it became clear that I overlooked the load cell's resolution. As noted by the R6 graph, the load cell is only able to measure forces applied within a 0.01N resolution. Meaning that, any force less than 0.01N failed to be registered by the sensor. In preliminary testing, however, the resolution didn't play a role in the overall data's integrity (as shown by graph R7). Therefore, a force of 0.012N was simply measured as 0.01N and a force of 0.004N was not measured at all. Upon further inspection, I realized that the design of the load cell platform was also inherently flawed.

The load cell is designed to measure the magnitude of applied forces as a function of "strain". As the applied force contorts the metal shape, the load cell is able to convert the continuous analog signals into discrete digital ones. Because of the foam bullet's low mass, it was not able to "strain" or "move" the load cell platform to the same extent as the 1kg mass. For this reason, the values reported back are, on average, the smallest possible value readable by the load cell. On a similar note, the foam bullet's impact occurs within 400ms meaning that the arduino is only able to capture ~4 'snapshots' of the impact. As a result of the load cell and arduino limitations, I performed 9 trials at each individual distance and calculated the average of each. The idea being that the average of each of the individual distances would be different enough to form a generalized model for FoI as a function of displacement.

Finding Uncertainties:
The three quantities measured are force of impact, time, and distance. Thus, uncertainties must be calculated for each of these.

Distance
Because the distance is measured with a meter stick, the uncertainty can be written as $\pm 0.05cm$.

$$\sigma_x = \frac{smallest\ increment}{2}$$
$$\sigma_x = \frac{1mm}{2} = 0.5mm = \pm 0.05cm$$

Uncertainty for Force
Much like distance, the observed minimum resolution of the load cell appears to be 0.01N. Therefore, the absolute uncertainty for the instantaneous force readings is $\pm 0.005N$.

$$\sigma_x = \frac{smallest\ increment}{2}$$
$$\sigma_x = \frac{0.01N}{2} = 0.005N = \pm 0.005N$$

Load Cell readings are also affected by a series of factors such as creep, ambient temperature, and calibration

In order to maintain the integrity of the data, the load cell was calibrated with a known mass of 1kg, and this was repeated prior to each round of data collection.

Uncertainty for Time
The Arduino Nano was observed to request data in 90ms intervals. Therefore, the time uncertainty can be calculated as such:

$$\sigma_x = \frac{smallest\ increment}{2}$$
$$\sigma_x = \frac{90ms}{2} = 45ms = \pm 45ms$$

Example Calculations (FoI Uncertainty)
As is noted from the results graph at the beginning of this section, individual uncertainties were calculated for each trial. The process is explained below.

For any individual trial, such as the one to the top right (R8), the absolute uncertainty was calculated using the individual measurement's relative uncertainty.

Time (ms)		Force (N)	
0	±0%	0.00	±0%
90	±50%	0.00	±0%
180	±25%	0.01	±50%
270	±17%	0.01	±50%
359	±13%	0.00	±0%
449	±10%	0.00	±0%

(R8 $F(t)$data for Trial 9 at 10cm)

Relative uncertainty can be found by dividing the absolute uncertainty by the 'best guess', which is the measured value, and casting as a percent. The relative uncertainty for each value is presented in the data table R8 to the right of its respective data.

When calculating the uncertainty for the theoretical FoI, the percent uncertainties for F_m, and d_i were added because the two values are being multiplied.

$F_m \rightarrow$	0.01N
$d_i \rightarrow$	449ms
$F_{avg} \rightarrow$	0.003N

Theoretical FoI Uncertainty

$$\int_0^{d_i} F(t)\,dt = \frac{2}{3}F_m d_i + C$$
$$\int_0^{d_i} F(t)\,dt = \frac{2}{3}(0.01N \pm 50\%)(449ms \pm 10\%)$$
$$\int_0^{d_i} F(t)\,dt = 2.993N \pm 60\% \approx 3.0\,N \pm 60\%$$

Actual FoI Uncertainty
Because the actual force of impact deals with manual integration of the $F(t)$ graph, the uncertainty is calculated as the following summation:

Equation for manual integration

$$\sum_{n=0}^{k=\frac{90}{d_i}} F(90n)90 \bullet (1 + \frac{F(90(n+1)) - 1}{2})$$

Where $F(90n)$and $F(90(n + 1))$ represent the load cell's force readings, and the 90 to the right of $F(90n)$ represents the width of the rectangle used to approximate the area.

Equation for Actual FoI Uncertainty

$$\sum_{n=0}^{k=\frac{90}{d_i}} F(90n)90 \bullet (1 + \frac{F(90(n+1)) - 1}{2})((\frac{0.05}{F(90n)} + 0.5) + \frac{0.0}{F(90(}$$

When calculating the actual FoI uncertainty, we are adding the absolute uncertainties of each 'mini' integration which requires the calculation of the relative uncertainties. The calculation for the relative uncertainties can be found below:

$$\sigma_x = ((\frac{0.05}{F(90n)} + 0.5) + \frac{0.005}{F(90(n+1)})$$

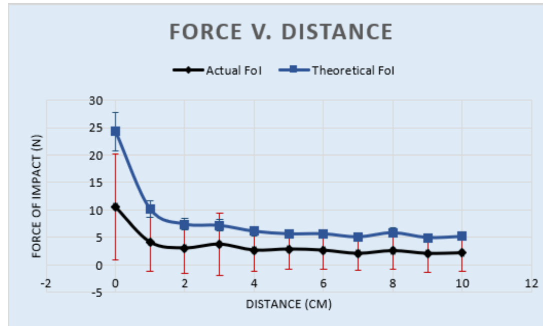
This is then multiplied by the integral between the two measured points. Like with the rest of the data, this is repeated for all 9 trials and then averaged in order to gain a proper understanding of the foam bullet's FoI, both theoretical and actual, per distance x.

Observations on Uncertainty

The uncertainties for most of the calculations were uncharacteristically high. Like I mentioned before, the load cell's low resolution combined with the arduino's low sampling rate meant that the data was expected to be very uncertain. As I performed the experiment and realized the extent of the uncertainty, I attempted to compensate by doing 9 trials instead of the original 5 I had planned.

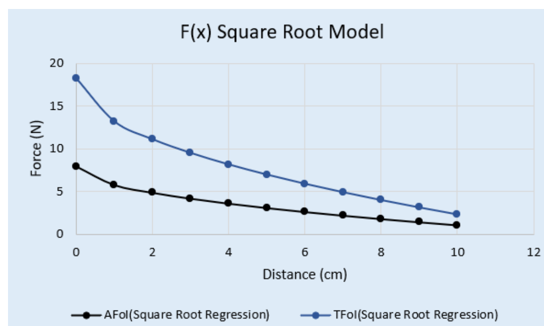
The Results

The $F(x)$ graph (R9) demonstrates the relationship between FoI and distance. Both the theoretical and actual FoIs follow a negative square root function and appear to be off by some factor L_c. The data table for the aforementioned $F(x)$ graph can be found either in the appendix or at the beginning of the Evaluation section (labelled as R2).



(R9 $F(x)$graph)

As mentioned, the actual FoI and theoretical FoI both appear to follow a square root function. Therefore, I performed a square root regression analysis on the data points to generate a line of best fit in the form of a square root function. Both lines are shown below:



(R10 $F(x)$ graph modelled as square root)

AFoI (Actual Force of Impact) Square Root Model ($R^2 = 0.73$)

$$AFoI(x) = -0.0561334\sqrt{1500x} + 7.9804$$

TFoI (Theoretical Force of Impact) Square Root Model ($R^2 = 0.73$)

$$TFoI(x) = -5.0247\sqrt{x} + 18.2404$$

Observations of Results

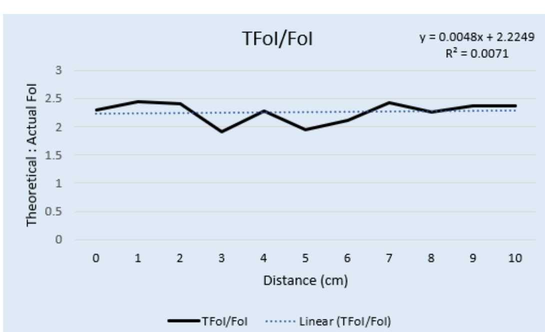
As distance, x, increases, there is a notable difference in the error, both absolute and relative, of the actual and theoretical models. Interestingly, however, as distance x increases, AFoI and TFoI appear to converge on each other.

This is to be expected, since as x approaches 0, so does the FoI; be it theoretical or actual. Furthermore, the discrepancy between the actual and theoretical model can be attributed to these 'negotiations' between the foam bullet and its environment. As the bullet exits the NERF gun's barrel, there is a loss of KE due to the friction between the foam and the barrel's inside wall. Moreover, as the foam bullet traverses the air, there is a force of drag applied to it that is somewhat proportional to v^2. In fact, there could be a very intimate relationship between Drag Force and Force of Impact. This would explain why after reaching "terminal velocity", both models appear to converge on each other.

Referring back to the theoretical model of FoI as a function of time,

$$\int_0^{d_i} F(t)\, dt = \frac{2}{3} F_m d_i$$

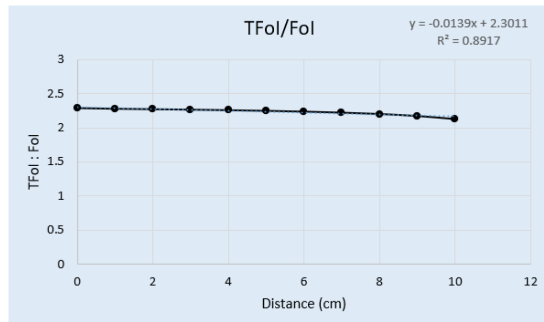
The model above assumes that the $F(t)$ graph will be symmetrical and follow a parabolic curve. It could be said, that the graphs' inability to follow the parabolic curve could be due to the energy losses from the air, the barrel's inside, etc. Observing graph R11, the ratio between TFoI and AFoI can be graphed in order to see if there are any patterns to better describe the force of impact as a function of time.



(R11 $TFoI/FoI$ per each distance)

The line of best fit of the ratio of TFoI and AFoI shows that the TFoI is always ≈ 2.225meaning that if you were to divide the TFoI by 2.225, the resultant FoI would be very close to the AFoI.

The same is done to the square root model of the $F(x)$ graph:



(R12 $TFoI/FoI$ per each distance)

The linear nature of figure R11 and R12 could be due to the idealized nature of the square root model, but it is important to recognize the similarity between the two ratios. The $F(x)$ graph's ratio appears to be an average of ≈ 2.3while the $F(t)$ graph appears to be, as mentioned before, ≈ 2.225.

Referring back to the initial question of: "At what rate does a foam bullet's force of impact change in respect to its total displacement from origin?". With the gathered data, the relationship between displacement and FoI is a negative square root as shown below:

$$TFoI(x) = -5.0247\sqrt{x} + 18.2404$$

Furthermore, the formula can be changed to account for the discrepancy between the observed FoI and theoretical calculations.

$$TFoI(x) = \frac{(-5.0247\sqrt{x} + 18.2404)}{2.3}$$

By dividing by 2.3, we are able to better approximate the FoI in relation to distance. One could also do the same for the initial theoretical FoI calculation $F(t)$:

$$\int_0^{d_i} F(t)\,dt = \frac{2}{(3\bullet 2.225)} F_m d_i \approx 0.29963 F_m d_i$$

It should be noted that used separately, these equations are proven beneficial in determining the FoI given at a specific distance or time when $0cm \le x \le 10cm$.

Conclusion

It is evident that there is a discrepancy between the theoretical and experimental calculations for Impact Force. Interestingly, the discrepancy between the two values is a constant factor of around "2.3"; or the loss constant L_c. L_c is significant because it allows us to accurately measure the AFoI at any distance from 0cm to 10cm. The original theoretical models failed to take into account losses of energy due to drag and friction, but by introducing L_c, we are able to account for such losses in energy. The model, however, fails to demonstrate the exact decreases in energy as a result of drag or friction. Follow-up experiments could be designed to better understand how different models of guns, and the coefficient of friction between the gun's barrel and the foam's bullet, affect the final impact force. Likewise, further analysis of the loss constant, L_c, could potentially be used to analyze the force of drag on a projectile. Doing so could lead to the creation of a more refined model for calculating average drag force exerted on an object.

The results found in this IA, however, should also be placed in the context of the uncertainties. Given the load cell's resolution and the arduino nano's low sampling rate, the model provides a generalized understanding of what occurs during the foam bullet's impact; which is a strength - but also a weakness. Further improvements for this lab include purchasing a load cell with higher resolution as well as investing in a microcontroller with a faster sampling rate. Given the current circumstances, doing so would have proven difficult. Furthermore, the model represents solid objects with low mass whose acceleration is powered by spring. Likewise, the foam bullet's shape most likely also plays an important role in determining FoI as well as the material (as was observed, larger d_i lead to lower FoI).

With these things in mind, I propose the following improvements to the lab:

(1) Purchasing Higher Sampling Rate Microcontroller. [Improves: High Time Uncertainty]
(2) Purchasing a load cell with a higher resolution. [Improves: High Force Reading Uncertainty]
(3) Changing the load cell's platform to a more a more sensitive / flexible materials such as a thin plastic or PLA-like material. [Improves: High Force Reading Uncertainty]
(4) Designing a better stand for the NERF gun, because the current one makes it difficult to always fire the bullet in the same location. [Improves: Reproducibility of the Experiment]

Testing the Model:

In order to test the model, I have provided a hypothetical scenario:

A haunted house wants to include an automated NERF gun that fires a foam bullet as a person turns a corner, but the organizers don't want to harm anyone. The organizers wonder at what distance, in cm, the FoI reaches zero.

In this scenario, the solution can be found by simply setting the theoretical formula $F(x)$ to zero and solving for x.

$$F(x) = 0$$
$$-5.0247\sqrt{x} + 18.2404 = 0$$
$$\sqrt{x} = \frac{-18.2404}{-5.0247}$$
$$x = \frac{-18.2404}{-5.0247}^2 = 13.17796777 \approx 13.2cm$$

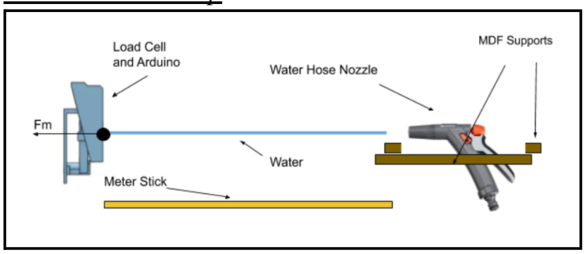
According to the calculation, the force of impact at distance 13.2cm should equal zero. I performed 9 trials at distance 13.2cm with a different gun than the one used to collect the data from the model.

The average FoI at 13.2 cm with a NERF Strongarm is $\approx 1.9N$. The last recorded distance, $10cm$, has an AFoI of $2.2 \pm 3.3N$. As expected, the FoI does decrease but it is not 0; as the model dictated it would be. In part, the NERF Strongarms' spring appears to be slightly stronger than the NERF Elite Firestorm (which was used to collect data for the model). Likewise, a FoI of 0, would imply that the foam bullet never makes contact with the object and perhaps a better measure of the distance necessary to achieve a force of impact of "0N" would be through the use of basic 2D kinematics.

This reinforces the idea that there is room for improvement, as mentioned prior to the hypothetical example. An additional area of improvement is to test with different NERF guns as opposed to only a singular one. This experiment, however, has prompted a series of questions regarding the nature of impacts.

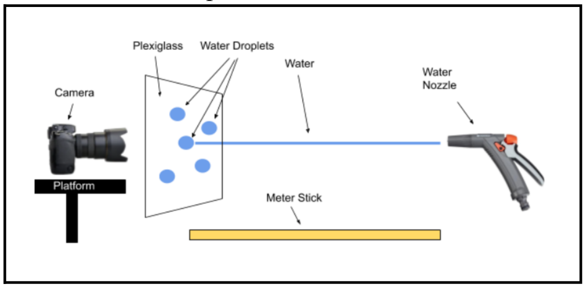
Does a liquid's FoI differ from a solid's FoI, if so, how?

Possible Lab Set-Up



To what extent does the distribution of mass affect the resultant force of impact of a projectile?

Possible Lab Set-Up



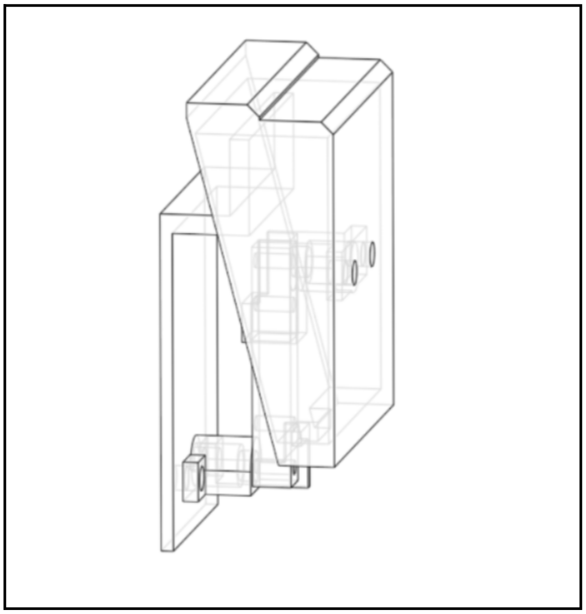
Having experimented with load cells in this IA, I foresee a couple of possible sources of error with the aforementioned experiments:

Timing of Water Hose and Constant Flow Rate

- Because of Impact Force's dependence on Δt, the timing of the water hose should be kept constant. Investing in motors or servos to actuate the water hose pulley would greatly help in creating reproducible results. Unfortunately, this involves additional expenses as well as programming beyond the scope of the experiment.
- Furthermore, the rate of change of pressure from the water hose (supplied by a displacement of the water hose's pulley) is also subject to human error. Once again, reasonable solutions include: motors or servos.

Interference of Water on Load Cell

- The water could cause fluctuations in the Load Cell's readings. I designed a case for the load cell to fit into and possibly protect it from water (shown below).



(R12 Design for Water Proofing Load Cell)

Looking back at my initial interest with NERF guns, it was nothing but a superficial interest based on the idea that the NERF gun was nothing more than a toy. Now, having explored this topic further, I find that the NERF gun is much more than what it appears; it is a tool which I can use to explore physics. Although my curiosity has been satisfied, I do wish to continue exploring the relationship between force of impact and range in a much more accurate manner. Likewise, this experience has led me to question how some of my other "toys" could be applied to the expansion of my understanding of physics.

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